BINARY OPINION DYNAMICS WITH MESOPHILIC AGENTS

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ABSTRACT

The study of opinion dynamics has experienced a gain in interest recently, especially with individual communication infrastructures like the Internet. Much of this research uses simulations with agents whose behavior and preferences are governed by a small set of rules. Most agent models in the literature are homophilic and/or conforming. In this work, we explore the opinion trajectory and self-sorting tendencies of a relatively new agent type: mesophilic agents, or those whose preference is for an even mix of similarity and difference of opinion with each neighbor. We characterize the behavior of these agents in topologically dynamic networks, and examine long-term behaviors of networks via common metrics as well as the reward function used to reflect agent preferences. We empirically uncover several apparent natural boundaries on things like network density and average reward score across agents that may lend themselves to a more analytical treatment moving forward.

Keywords: network science, agent-based, emergent behavior, multiagent systems.

1 INTRODUCTION

In various subfields of network science, particularly self-organization and opinion dynamics, the overwhelmingly most common agent modeling paradigm is based on *homophily* – that is, agents prefer connections to others who are more similar to themselves to those who are less similar, however "similarity" is defined in the context at hand (McPherson, Smith-Lovin, and Cook 2001, Gargiulo and Gandica 2016). Often these agents will also be *conforming*, or have a behavioral primitive that causes them to become more similar to their neighbors over time. These two aspects are used separately or in conjunction in most research in order to model typical human preferences and behavior, and make up the dimensions of what we refer to as *homophily/conformity space* (HC), described in Figure 1. For the remainder of this paper, we will refer to agents that are both homophilic (more similarity with neighbors is always preferred to less) and conforming as *classical* (C) agents.

Although most research models are homogeneous in agent type, using classical agents as the model, this is clearly not a robust representation of human behavior. To begin filling this gap, other researchers have proposed, e.g., heterophilic agent models (Motsch and Tadmor 2014) that fall on the left extreme of the homophily dimension, as well as contrarian agents (Galam 2004) that adopt the minority opinion in a network or in their neighborhood, thus actively upsetting the process of consensus formation. Both of these models have immense theoretical value, but fail to account for non-extremal nuance in agent preferences. In the real world, there are very few people who prefer exclusive similarity or dissimilarity, instead finding their maximal sense of personal value from a diversity of experience. The initiatives to diversify the campus community, workplace, and other demographics are one striking example. When compared to "colorblind" statements that recognize our common humanity, it has been discovered that the value of human *differences* conveys a multicultural message with positive results (Carnes, Fine, and Sheridan 2019). This work fills a gap in the current corpus of network science knowledge by exploring the behavior of agents whose maximal sense of value is produced by having neighbors that are exactly half “similar" and half “dissimilar" to themselves. We refer to these agents as *mesophilic*.

In addition to agent preferences, opinion trajectories are equally important to understand the evolution of influences on agents’ satisfaction over time. For this purpose, we use two different models of opinion trajectory: *conforming*, by which agents gravitate toward the majority opinion within their neighborhood over time; and *rebelling*, choosing instead to move away from their neighborhood’s majority opinion, as these are the two most common trajectory models in use today.

2 RELATED WORK

Perhaps the closest work in spirit to our own is a recent piece by Cheon and Morimoto (Cheon and Morimoto 2016), which incorporates classical agents called “majority rule floaters;" “inflexibles," or stubborn agents whose opinion remains constant throughout simulations; and “balancers," or agents who act as floaters when there are no inflexibles in their neighborhood, and who take on the opposite opinion of the inflexibles if they are present. This work combines three agent types, similarly to (Shepherd, Weaver, and Goldsmith 2020); in both, agents prefer a sense of balance.

Heterophily, or the tendency of agents to interact with others dissimilar to themselves, is as yet a less well-studied phenomenon in terms of social systems than its more prevalent counterpart: homophily. The canonical example in this area is the classic Schelling model of segregation, in which agents have a threshold of difference they are willing to tolerate before changing their position within the network (Schelling 1971). Manzo and Baldassarri explore the role of heterophily in a status/deference-based society (Manzo and Baldassarri 2015), while Picascia and Mitchell use it to investigate fairness in green space usage (Picascia and Mitchell 2022). Trogdon and Allaire used heterophily across multiple “friendship characteristics" to implement a friend selection model in order to study the effect of agents’ choices of social connection on obesity (Trogdon and Allaire 2014). Kohne *et al.* look at how homophily and heterophily affect the development of conflicting social norms (Kohne, Gallagher, Kirgil, Paolillo, Padmos, and Karimi 2020). Neal and Neal use homophily and heterophily to speculate on the competing nature of the values of diversity and community (Neal and Neal 2016). Motsch and Tamdor illustrate the effects of heterophilous agents on forming and maintaining consensus (Motsch and Tadmor 2014).

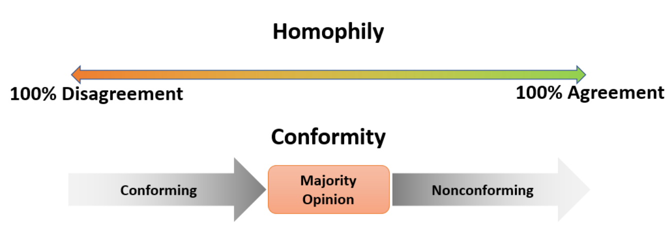

Figure 1: Homophily/Conformity Space.

It is mostly the non-typical reward function that sets our agents apart from others in the literature, but as demonstrated a great deal of the literature dealing with heterophilic dynamics relates only loosely to opinion diffusion. Several other bodies of research deal with non-standard agent models. Possibly the best known of these alternate agent types is the *contrarian*, a notion that has been explored by several researchers but most heavily popularized by Galam (Borghesi and Galam 2006, Galam 2004, Galam 2013). In this work we incorporate a concept of an agent that is a *minoritarian*, preferring to always change their own opinion to match that of the minority of their neighborhood rather than the majority. Other examples of well-known variations on agent modeling include attractive-repulsive forces that not only govern what an agent's opinion will move toward, but also away from (Martins, Pineda, and Toral 2010, Milli 2021, Proskurnikov, Matveev, and Cao 2015); skeptical agents (Monica and Bergenti 2017, Tsang and Larson 2014); and separate public and private opinions, thus requiring agents to have a mechanism governing their degree of honesty (Botan, Grandi, and Perrussel 2019, Hansen and Ghrist 2021, Hou, Li, and Jiang 2021).

The purpose of this work is to provide detail and insight into a region of agent modeling space that has not yet been well explored: agents with a single-peaked, non-monotonic reward function, in versions that behave as majoritarians as well as contrarians. By examining dynamic networks, we can shed light on the opinion dynamics in isolation of these agent types, as well as how the opinion dynamics govern topological interplay when agents are allowed to self-organize while their opinions evolve.

3 MODELING METHODOLOGY

We use a typical binary opinion diffusion model in which agents can have one of two opinions on a number of categories, and they change their opinions over time based on the opinions of their neighbors. In addition, agents have preferences over the similarity of their neighbors – if they are dissatisfied with a neighbor's opinions, they can disconnect themselves from that neighbor. This section lays out the notation we will use to represent these characteristics, and describes the individual mechanisms implemented to simulate them.

3.1 Notation

Our work focuses on the evolution of a social network represented as an unweighted graph $G = \langle V, E \rangle$ where V is a finite set of agents, and $E = \{(u,v) : u, v \in V\}$ is the set of social links between them. Each agent i in our networks has a k-dimensional binary opinion vector $\mathbf{x}_i^t = \{-1, 1\}^k$ that evolves over time, hence the superscript t. Except where necessary for clarity, the superscript t will be omitted. Given some set of agents $\mathbf{S} \subseteq V$, we can define $\mathbf{X}_{\mathbf{S}}^t = \{\mathbf{x}_i^t : i \in \mathbf{S}\}$ as the *opinion profile* of agents in $\mathbf{S}$ at time t. Agents update their opinions based on the opinions of their neighbors. We denote the *neighborhood* of i as $N(i) = \{j : (i, j) \in E\}$. In addition, in this work we will at times assume that $\forall i : i \in N(i)$. In any other case, we will use $N(i)_{-i} = \{j : (i, j) \in E \wedge i \neq j\}$ to denote the neighborhood of i with i itself excluded. For convenience, we will use $upd(N(i))$ to refer to the rule by which agents update their opinions based on their neighborhood including themselves, and $upd(N(i)_{-i})$ when only considering their neighbors' opinions.

3.2 Agent Preferences

Generally, agents have a reward function that increases monotonically with the similarity between the focal agent and its target (either neighbor or neighborhood). Since our agents' preferences are defined on opinion-space, we utilize the following construction for reward functions. Let $H(\mathbf{x}_i, \mathbf{x}_j)$ be the traditional Hamming distance between two opinion vectors $\mathbf{x}_i$ and $\mathbf{x}_i$ of equal length k – that is, the number of element-wise differences between the vectors. We normalize this figure to produce an opinion distance defined as:

$$d(x_i, x_j) = \frac{H(x_i, x_j)}{k}$$

or the percentage of disagreeing entries. Then respectively, the reward functions used by strictly *homophilic* and *heterophilic* agents i to evaluate their approval of the opinions of neighbor j are $r_{hom}(i, j) = 1 - d(x_i, x_j)$ and $r_{het}(i, j) = d(x_i, x_j)$. In this work, we focus mainly on *mesophilic* agents that prefer neither strict agreement nor disagreement when considering their neighbors' opinions. For these agents satisfaction with a neighbor is maximized when the agent shares the same opinion as their neighbor on exactly half of the available topics. To reflect this, a mesophilic agent i uses the following reward function to gauge its satisfaction with neighbor j:

$$r_{mes}(i, j) = 1 - (2|d(x_i, x_j) - 0.5|)$$

This reward function satisfies our desired properties, reaching its maximum of 1 at a distance of 1/2, and dropping to 0 as the distance approaches its maximum (total disagreement) and minimum (total agreement) values. The role that this reward function plays in our work is to govern which connections persist and which are eliminated. Much as with the classical Schelling model (Schelling 1971), agents are allowed to change their position within the network in terms of their neighbors. In our experiments, agent i is allowed to "unfriend" a neighbor j if $r_{mes}(i, j) < 0.5$ with some probability defined as a parameter for the simulation.

3.3 Conformity

All opinion diffusion research involves a model in which agent opinions change over time. There are several canonical models that are generally used in the literature, as well as innumerable extensions of each. We use a k-dimensional binary opinion vector for each agent. Each agent follows the *majority rule* when updating its opinions – that is, if strictly greater than half of i's neighbors have a certain opinion in a given dimension, then i will flip its opinion on the topic.

In the present work, we investigate the behavior of agents that follow one of two update rules: **conforming** (update to *agree* with the majority of neighbors) and **rebelling** (update to *disagree* with the majority of neighbors). To find the opinion leanings of agent i's neighborhood, we calculate the column average of its opinion profile:

$$\overline{\mathbf{X}}^t_{N(i)} = \mathbf{W}\mathbf{X}^t_{N(i)}$$

where $\mathbf{W}$ is a square matrix of size $|N(i)| \times |N(i)|$ with all entries equal to $1/|N(i)|$, with the neighborhood average opinion on topic k being denoted $\overline{\mathbf{X}}^t_{N(i)}(k)$. We also use a construction in which i excludes itself from its local average opinion, in which case $N(i)$ must be replaced with $N(i)_{-i}$. Using this, we apply the following update rule to each opinion $x_{ik} \in \mathbf{x}_i$ for conforming agents:

$$x^{t+1}_{ik} = \begin{cases} -x^t_{ik} & x^t_{ik} * \overline{\mathbf{X}}^t_{N(i)}(k) < 0 \\ x^t_{ik} & \text{otherwise} \end{cases}$$

If a conforming agent's neighborhood has an average opinion that disagrees with its own, then it flips its opinion; otherwise, it remains the same through the next step. Rebelling agents' opinions follow a similar rule, but the flip is in the opposite direction:

$$x^{t+1}_{ik} = \begin{cases} x^t_{ik} & x^t_{ik} * \overline{\mathbf{X}}^t_{N(i)}(k) < 0 \\ -x^t_{ik} & \text{otherwise} \end{cases}$$

Rebelling agents actively fight against consensus, always moving their own opinion to align with their neighborhood's minority. If an agent's neighborhood is evenly split on a topic, then that agent retains its original opinion. The combination of reward function/update rule is sufficient to define our agents and how they will behave. Thus, we have two agent types under investigation: *mesoconforming* (MC) and *mesorebelling* (MR); each uses a mesophilic reward function, and one type conforms to its neighborhood opinions while the other rebels.

3.4 Simulation Environment

Our experiments were meant to characterize the behavior of these agents in topologically dynamic networks. For the latter case, we intended to probe the effects of three network parameters on the agents' overall behavioral patterns: 1) the balance of friending and unfriending probability; 2) the balance of proportional representation of agent types; and 3) the differences between using $\overline{\mathbf{X}}_{N(i)}$ and $\overline{\mathbf{X}}_{N(i)_{-i}}$.

We examine the behavior of homogeneous and dual-type networks, considering pairwise combinations of our agent types and the classical agent model. To control the flow of connections in dynamic networks, we provide two parameters $\Pr(+)$ and $\Pr(-)$. At each time step, every node is presented with a single new neighbor $j \notin N(i)$, and the edge (i, j) is created with probability $\Pr(+)$. Also at each time step, every node is allowed to choose to sever any connections to neighbors it chooses to. At every time step, every node i will sever its connection to any neighbor j such that $r_{mes}(i, j) < 0.5$ with probability $\Pr(-)$. The proportions of agent types are each controlled by individual parameters: p_{MC}, p_{MR}, and p_C such that $p_{MC} + p_{MR} + p_C = 1$. In this investigation, we only allow at most two of these parameters to be nonzero at a time.

4 EXPERIMENTAL RESULTS

In this section, we first briefly enumerate the specific parameters we used in our simulations. We then describe our observations for both homogeneous and dual-type networks, illustrating trends from networks under different parameterizations. Unless explicitly mentioned otherwise, all results presented in this section are either individual observations or the averages thereof over 10 independent runs.

All tests are performed with $|V| = 100$ for 1000 time steps. For dynamic networks, we examine the effects of balancing the probability of acquiring a new friend and that of severing the connection to an existing friend. We ran experiments with $\Pr(+) = 0.75 > \Pr(-) = 0.25$, $\Pr(+) = \Pr(-) = 0.5$, and $\Pr(+) = 0.25 < \Pr(-) = 0.75$. All of our dynamic networks are generated as R networks, because the nodes' self organization makes the initial topology meaningless. Homogeneous networks comprise only one agent type. We also perform experiments on each pair of agent types examined in this paper in different proportions. For every pair of agent types, we simulate networks that are made up 50% of each agent type, as well as both versions of networks with 25% of one agent type and 75% of the other.

4.1 Homogeneous Networks

Since C agents are homophilic, they will gravitate towards others of similar opinions over time given the opportunity. Taken together with the agents' tendency to form consensus implies that a topologically dynamic network with only C agents will eventually form a strict consensus and approach a density of 1, which is exactly the behavior we observe. Mesophilic agents' behavior is highly sensitive to the balance of $\Pr(+)$ and $\Pr(-)$. We tested different combinations of these two values, both when updating opinions using $upd(N(i))$ and $upd(N(i)_{-i})$.

MC networks using $\Pr(+) = 0.25, \Pr(-) = 0.75$ for topological dynamics and $upd(N(i)_{-i})$ for updates all converged to a density of 50%. In most simulations, agents settled into an equilibrium in which all agents stabilized on three topics, and oscillated on the fourth. In the other cases agents found an equilibrium by stabilizing on half the topics and oscillating on the rest. Since our agents are designed to sever a connection to a neighbor *only* when they get $< .5$ reward from that connection, two neighbors who agree and disagree on half the topics will get 1 reward per step from each other. When they flip their disagreeing opinions, they do so symmetrically and so stabilize into that cycle. Because there are four opinion dimensions, there are five possible oscillating states including those seen above: a state in which agents oscillate on zero topics and all topics stabilize, one in which they oscillate on one topic and the other three are stable, on in which they oscillate on two and the other two are stable, etc. For brevity, we will refer to these equilibria as *Osc0*, *Osc1*, *Osc2*, *Osc3*, and *Osc4*. Setting $\Pr(+) = \Pr(-) = 0.5$ produces much the same result, but these probabilities induced a much stronger tendency towards *Osc1*, with almost no instances converging to *Osc2*. Finally, setting $\Pr(+) = 0.75, \Pr(-) = 0.25$ appears to pass a critical value past which it becomes more likely that agents will sink into *Osc0*. This is the worst-case scenario for MC agents in terms of r_{mes} because it is generally the result of opinion consensus. If these networks reach a state in which every agent agrees on every topic, then there is no longer an endogenous source of opinion evolution and agents can no longer receive any reward from each other, and so separate completely. Ironically, it is an increased propensity for accepting new opinion influencers and reluctance to let go of old ones that causes these networks to become fragmented and desolate. Tests identical to those above but using $\overline{\mathbf{X}}_{N(i)}$ for opinion updates produce somewhat different results that follow the same trend. Agents are not able to reach an average value of r_{mes} greater than 0.5 in these instances, in part because their opinions agree with themselves, and thus are detrimental to their own reward score. Here again situations in which $\Pr(+) \leq \Pr(-)$ give agents the most latitude to gain more reward, increasing the probability that the network settles into *Osc1*. When $\Pr(+) > \Pr(-)$, the move to *0sc0* forms quickly and locks agents out of further progress.

MR networks still exhibit oscillating opinions, but in the dynamic setting these agents tend to organize themselves over time in a way that causes the density of the graphs to converge to approximately 48% in almost all cases when $\Pr(+) \neq \Pr(-)$. Using $upd(N(i)_{-i})$ and setting $\Pr(+) < \Pr(-)$ we again observed consistent equilibria that were by and large satisfactory for the agents; networks settled into *Osc2* or *Osc3* in all cases, providing all agents 1 or 0.5 reward, respectively. Conversely setting $\Pr(+) > \Pr(-)$ led nearly all networks to converge to *Osc3*, with one exception in which the network became entirely disjoint. It is possible that this network would, given enough time, transition to *Osc3*. However, in a situation in which all agents become separated they will simply swap all of their opinions every time step. If all agents are in one of two opinion states at all times, then every agent will stand to gain 0 reward from every potential neighbor, since all opinion sets are exactly opposite, and so a persistent state of disconnection can arise in rare instances. Setting $\Pr(+) = \Pr(-)$ introduced an interesting gray area. Here, many networks still settled around 50% density, but there were also those that went as high as 75% and several equilibria in between. All of these networks gravitated to *Osc1*, *Osc2*, or *Osc3*. Whereas in the other cases topological equilibrium states seem strictly quantized, this setting with the newly available spectrum of density thresholds also produces a broader set of possible long-term rewards and allows in more structural variety among possible long-term realizations of the network. The use of $upd(N(i))$ causes a much stricter quantization of these effects, no longer inducing the sort of topological variety we observed previously. Setting $\Pr(+) \leq \Pr(-)$ is largely positive for MR agents just like their MC counterparts. Under these parameter settings, all networks converge to a density of $\approx 50\%$, and opinion patterns settle reliably into *Osc2* or *Osc3*, again leading agents to a long-term reward of 0.5 or 1.0 per time step. Setting $\Pr(+) > \Pr(-)$ in these networks also passes a critical value. Some instances saw agents settling into *Osc2* or *Osc3* again, but in almost all cases these networks devolved quickly into *Osc4* which leads the networks to again become disjoint and stabilize into achieving 0 reward across time, as shown in Figure 2.

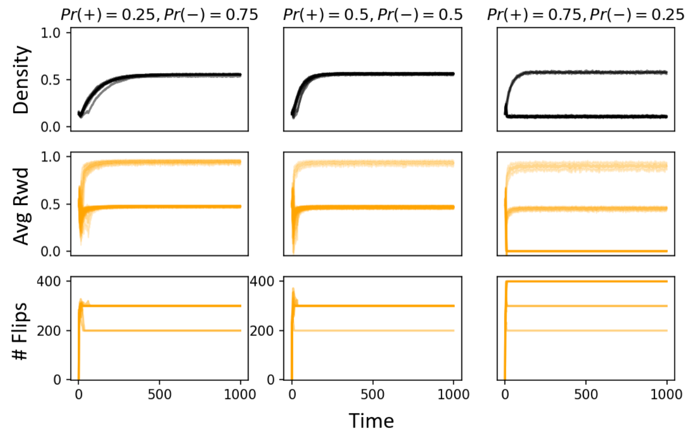

Figure 2: Summary of mesorebelling network metrics.

4.2 Dual-type Networks

Here we discuss results observed when simulating networks of two types of agents. We ran tests on all pairwise combinations of agent types, using the same conditions as above (both update rules, variable probabilities for $\Pr(+)$ and $\Pr(-)$), and we also varied the proportions of each agent type. For every type pair, we simulated networks with 50% of their agents from each type, as well as both combinations of 25% one type and 75% the other.

Dynamic C/MC Networks: We examined networks made up of half *C* and half *MC* agents, both conforming, updating their opinions based on $upd(N(i)_{-i})$. In these networks, *C* agents largely control the flow of evolution when they comprise at least half of the network. This is because *MC* agents have an opinion trajectory that is at odds with their reward function – they move towards total agreement, but prefer some disagreement. Therefore, these agents tend to end up helping to form an opinion consensus across the network when $\Pr(+)$ is high relative to $\Pr(-)$ as seen before, and therefore move to a place where $r_{mes} \approx 0$ across time. Conversely, *C* agents prefer as much agreement as possible, and so tend to maximize r_{hom}. This leaves the collection of friendships within the graph almost exclusively to those between two *C* agents, as *MC* agents lose their connections. As $\Pr(+)$ decreases and $\Pr(-)$ increases, we see more robust self-organization patterns. Here, we see two equilibrium states: one that mirrors the previous case in which MC agents get locked out and C agents reap all the benefit, and much more commonly a situation involving much more intermixing between types. We do observe across conditions that a given agent type typically self-organizes to be connected to mostly others of the same type, and that is true here as well. In these cases with numerous cross-type relationships, C agents still sink into *Osc0* where they prefer to be, but all MC agents come to a consensus on three topics and are split evenly on the last, oscillating between opinions at every step. This allows MC agents to settle into 0.5 reward per step despite the opinion trajectory of the network. Using $upd(N(i))$ eliminates MC agents' prospects for progress; MC agents always suffer because their consideration of their own opinion when updating changes the nature of the opinion flips they execute. MC agents always move to *Osc0* and gain 0 reward.

MC agents fare no better when they are a minority of the network and C agents are the majority. All of the trends listed above are even more pronounced in these cases, with a lone exception. Following from the patterns we have illustrated thus far, setting $\Pr(-) > \Pr(+)$ allows MC agents to be more discerning in who does and does not influence their opinions, and so in roughly half of our simulations MC agents were able to avoid the *Osc0* trap, and instead create a consensus around three topics and flip-flop on the fourth. This pattern always involves agents “trading opinions" on a single topic at every time step, and most of the

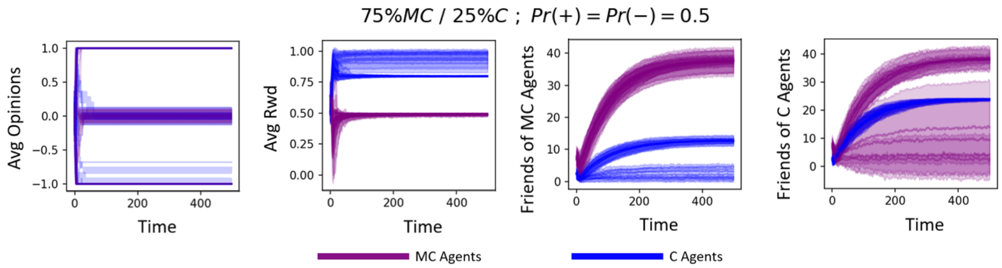

Figure 3: Mesoconforming agents in the majority find an equilibrium with classic agents.

interactions MC agents are a part of involve other MC agents, although there is some mixing. Finally, when MC agents comprise the majority of the network and C agents are a minority, we observe the highest level of variation among outcomes. Figure 3 illustrates some of these outcomes. An MC agent majority is able to overpower the consensus formation tendency inherent in the C agent type on some opinions, thus forcing an equilibrium that would not otherwise occur. The top left panel of the figure shows average opinion on each topic separated by agent type. If all agents in the network have opinion -1 on a topic, then the line corresponding to it will be at $y = -1$ on the graph; if half of the agents have opinion 1 and half have opinion -1 on a topic, then its line will fall along $y = 0$. It can be seen that often C agents will be kept away from their favored state of consensus, but this new equilibrium is not overly detrimental to C agents in terms of r_{hom} – they still perform well, often maximizing their own reward, but rarely dropping below 0.8 as shown in the top right panel. MC agents, in return for this cost to their minority counterparts, are able to settle into a state of achieving 0.5 reward consistently which is an outcome that is impossible for these agents under most conditions. The bottom two panels show the distribution of neighbors by type: the left panel shows (in purple) how many MC neighbors and (in blue) how many C neighbors each MC agent has, and the right panel shows the same information but for C agents. Under almost all conditions we examined, C agents only connected with one another, and whenever MC agents were able to make connections they were usually with others of the same type. In this situation, though, each type acquires a significant number of neighbors of the other type, showing a self-realizing stable equilibrium that is beneficial for both agent types but avoids the pitfalls of universal consensus. As in every case, $\Pr(-) < \Pr(+)$ is the worst for MC agents; in all but one simulation, all agents went to *Osc0* and remained there. In the exceptional case, it appears that a cohort of MC and C agents were able to form a stable subnetwork in which the MC agents induced opinion-flipping among the C agents – something that is very rare. In these cases, C agents' reward dropped from 1 to about 0.8, and MC agents' reward rose from 0 to 0.5; this rare circumstance does cost some reward for a portion of the network, but increases the aggregate satisfaction within the network as a whole dramatically. Letting $\Pr(-) \geq \Pr(+)$ causes our exceptional case to become the norm. When C agents are the majority, they tend to clump together, and MC agents have a difficult time gaining any purchase in their neighborhoods. Conversely, when C agents are the minority, we see networks becoming denser overall (although these numbers still converge to a long-term equilibrium < 1), caused by a much stronger intermixing of types within neighborhoods. The influence of this mixing causes more opinion variation among C agents, whereas when they made up a larger portion of the network they had no need to vary. As before, MC agents generally allow the network to form a consensus across multiple topics and then swap opinions between each other for the last one; this allows them to organize the network in such a way that allows them to achieve .5 reward at each time step, something they were barred from under other conditions. Using $upd(N(i))$ eliminates these possibilities, putting our agents in the same position they experienced under our other parameter settings that had shown promise before.

Dynamic C/MR Networks: Networks created with $p_C = p_{MR} = 0.5$ where agents use $upd(N(i)_{-i})$ show similar behavior but also illustrate some that is counterintuitive. In these networks, MR agents do not suffer as much as their MC counterparts from a value of $\Pr(+)$ that is high relative to $\Pr(-)$. MR agents show a

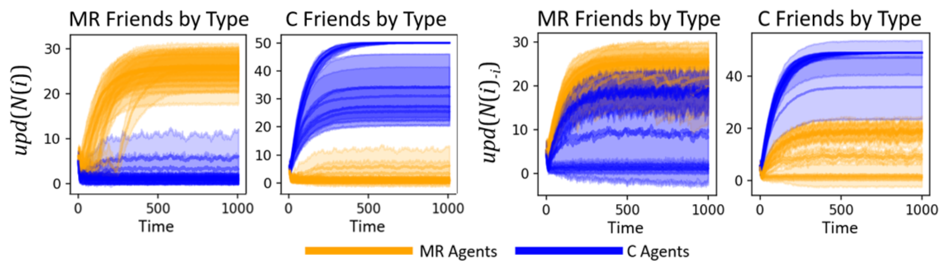

Figure 4: Mesorebelling agents and classic agents' friending patterns become more or less pronounced based on the update rule used.

slight preference for each other in terms of neighbors, and C agents still have a strong preference for their own type. MR agents generally converge to an opinion trajectory that is stable on at least two topics, and agents trade opinions on the others, either in whole or in part. Given the nature of r_{mes}, this is beneficial because agents typically achieve ≥ 0.4 reward per step, and depending on the particular situation can come close to maximizing reward across agents of their type. These networks seem to quickly approach a limiting density of 40%. As we allow for $\Pr(-) \geq \Pr(+)$, agents of both types do appear to prefer connections to their own type more strongly, although they still allow for a good amount of variation. C agents still do very well in terms of reward in these networks, typically never dropping below an average of 0.8, and MR agents also perform well generally averaging a reward of ≥ 0.5 – slightly better than when $\Pr(+)$ was set high. Unlike MC agents, MR agents are not able to induce much if any opinion variability into the C agent population. Using $upd(N(i))$ instead sees many of the same patterns arise, but MR agents' reward is limited to roughly 0.5, except in rare circumstances. Figure 4 shows the effect of the choice of update rule on connection tendencies between agent types, with $upd(N(i))$ causing much stronger type segregation.

When $p_C = 0.75, p_{MR} = 0.25$, MR agents are beholden to the whims of the C agent majority. With $\Pr(+) = 0.75$, MR agents are nearly unable to capitalize on their own reward function. They typically gravitate towards mostly C neighbors rather than their own type, and their total opinion oscillation drops to near-zero in most circumstances. The densities of these graphs appear to have two possible equilibria: one around 60% and the other around 30%. When C agents make up the majority and $\Pr(+)$ set high, the latter case rarely manifests as it represents instances in which the C cohort becomes split between opinion poles and thus divides topologically as well. As $\Pr(-)$ grows, MR agents' reward acquisition comes much more easily. The possibility of a long-term zero-reward configuration still exists under these settings, but it is much more rare. Generally, C agents are always able to maximize r_{hom}, and MR agents fall into the 0.5 or 1 reward equilibrium by only oscillating opinions as much as necessary. Interestingly, due to the repulsive nature of MR agents' opinion updates, these networks remain open to jumping between equilibrium states, and our observations imply that these jumps tend to be from a state that is less advantageous to one that is more, in terms of reward. When the majority of agents in the network are the MR type, C agents have more trouble achieving high satisfaction over time. Even though in most contexts C agents quickly settle into *Osc0*, the outnumbering presence of MR agents makes this nearly impossible. One benefit of MR agents is that they have very few conditions in which they do not regularly change their opinions, which in networks with stochastic connections can be a regular source of new opinion influences. In this context, they test how the C cohort behaves in suboptimal conditions. When $\Pr(+) = 0.75$, C agents still clump together as usual and experience very little opinion variation. They do still admit MR neighbors over time, although it is common to see the groups mostly separate. Setting $\Pr(-) = 0.75$ makes it difficult for C agents to connect to one another because the MR opinion influence keeps their opinion profiles at odds with one another. C agents do still have a large proportion of C neighbors on average, but also intermix heavily with MR agents.

Dynamic MC/MR Networks: Finally, we examined networks made up of both mesophilic types. When we observed homogeneous networks, we saw that each of our agent types has a characteristic pattern of

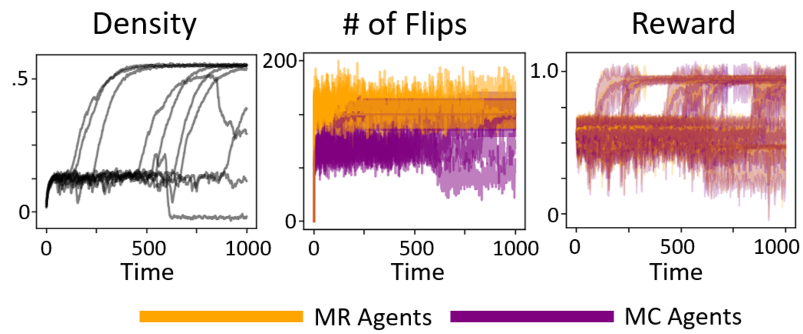

Figure 5: MC and MR agents in combination. MC agent data is purple, MR data is orange.

behavior – MC agents tend toward consensus and often become disconnected, MR agents tend to flip-flop opinions and be more adept at acquiring and keeping connections. When comprising an entire network together, these agents often work synergistically, although it is sometimes difficult for them to find a place of mutually beneficial equilibrium. It does appear in most cases that these agents gravitate toward a self-reinforcing norm condition given enough time, and provided that the network does not enter some sort of a sink state that bars further agent progress.

First, setting $p_{MR} = p_{MC} = 0.5$ and updating based on $upd(N(i))$ we observed some familiar trends. Most starkly apparent is the networks' tendency toward 50% density. Almost all parameter settings lead to this outcome indicating that mesophilic agents, when given the opportunity to reorganize themselves based on preference, will tend to find a self-stabilizing topological equilibrium. In many cases, networks can take a significant amount of time to settle into these stable states. In some simulations, networks would hover around a very low density for only around 100 steps before some critical condition is met and a quick transition into the final 50% density state is made. In others, the transition does not happen until closer to 750 steps. It stands to reason that other simulations that did not reach this state may still have done so given more time. Another rare possibility for these networks is that they settle into a minimal density state after some number of time steps – the density of these networks never goes entirely to 0 because of the stochastic connection process, but no new connections are kept for long. What we did *not* observe were any instances in which a network reached its 50% density equilibrium and then subsequently leave it. This self-stabilizing mechanism appears beneficial for both agent types as they both regularly acquire an average of at least 0.5 reward per step, and instances that tended toward 50% density saw both agent types maximizing their reward. An interesting point about this is that agents no longer find themselves restricted to forming an unwavering stable opinion profile like C agents do, nor do they tend towards an oscillation between two opposite opinion poles. Instead, opinions change much more organically over time in many cases, and in many others still behave cyclically but in a more complex manner. For instance, in multiple simulations a large cohort of agents would fall into an opinion cycle of length 4. These trends can be seen in Figure 5. The figure shows results from a set of 10 simulations, almost all of which at some point transitioned very quickly to their long-term equilibrium, the effects of which are reflected in the reward scores. Interestingly, the average number of opinion flips across time tends to vary even when an equilibrium state has been reached, and so these stable conditions appear to be maintained more or less organically. It can clearly be seen that in entirely mesophilic networks the notions of *Osc0*, *Osc1*, etc. no longer apply. Interestingly, using $upd(N(i))$ in these networks produces the sort of quantized results we saw in other situations – e.g., agents preferred their own type more strongly, and agent reward stabilized at either the 0.5 or 1.0 levels with little of the variation seen in Figure 5. When we unbalanced the proportions of agent types, many trends remained the same. Overall, MR agents appear to be much more easily satisfied by their network conditions in a variety of settings, and this is no exception. In these networks the use of $upd(N(i)_{-i})$ produces patterns closer to the *Osc* patterns mentioned earlier, although they can fall into numerous in-between categories as well (e.g.

stabilizing on two opinions and then half or three-quarters of the agents swapping back and forth). It is the inclusion of one's own opinion during the update that causes a significant amount of topological disruption.

5 CONCLUSION AND FUTURE WORK

In this work we have extended investigations into mesophilic agents in dynamic networks. We examined equilibrium states (and their absence) under a variety of different conditions pertaining to the topological composition and evolution of the relevant networks as well as the agent-specific state update rules. Our major findings are the following: dynamic networks including mesophilic agents of either type experience a hard limit on their density, regardless of the probabilities that nodes will become connected or disconnected; mesophilic agents in a multidimensional opinion space will tend to settle with their neighbors into a pattern of opinion change or stability that suits their reward function, and are rarely unable to do so; MR agents tend to be easier to satisfy in terms of their reward function, and also act as a catalyst for opinion evolution wheres MC agents are catalysts for consensus; the incorporation (or not) of an agent's own opinion into its updated opinion has dramatic effects on the topological evolution of dynamic networks; when agents' likelihood of severing an undesirable connection is higher than their probability of accepting a new one, the agents tend to be more satisfied on average. The final point above is an important observation not just for the design of opinion diffusion simulations, but also for real life – when our agents were too quick to accept new neighbors and not quick enough to jettison bad ones, those agents suffered for it. Protecting their own interests in terms of neighbors tended not only to be beneficial for the agents themselves, but also for the network as a whole as it fostered more opinion diversity. A clear next step of this work must be to develop network parameterizations that conform to real life social networks closely, and then verify that our model reflects genuine social phenomena in a realistic way. The application of such a model to voting situations could provide a greater ability to predict *a priori* and explain retrospectively voter behavior.

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